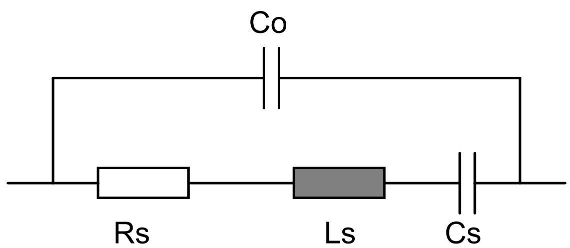
Measurement of the equivalent circuit of quartz crystals

This application note shows how to measure the equivalent circuit of a quartz crystal with Bode 100.

A.) Basics:
An equivalent describtion of a quartz crystal is given by the following circuit. It is valid in the region of a single Series-Parallelresonance combination. As can be shown these combinations occur at odd multiples of the fundamental Series resonant frequency.



With known values of L_S, C_S, C_0 and R_S we obtain the following equations:

$$\text{Series resonant frequency:} \quad fs = \frac{1}{2 \cdot \pi \cdot \sqrt{Ls \cdot Cs}}$$

$$\text{Parallel resonant frequency:} \quad fp = fs \cdot \sqrt{1 + Cs / Co} \approx fs \cdot \left(1 + \frac{Cs}{2 \cdot Co}\right)$$

$$\text{Quality factor at } fs: \quad Q = \frac{2 \cdot \pi \cdot fs}{Rs}$$

As we don't know these values we have to solve the equations for L_S, C_S first, because we can measure C_0, f_S and f_P with Bode 100.

$$Cs = \left(\frac{fp}{fs} - 1\right) \cdot 2 \cdot Co$$

$$Ls = \frac{1}{4\pi^2 fs^2 Cs}$$

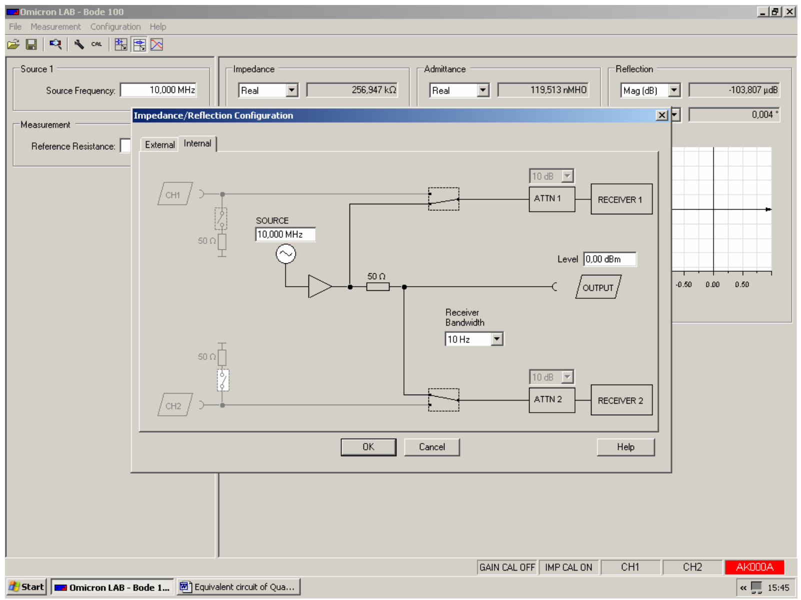
B.) The Device under Test is the crystal mounted at the Test PCB delivered with Bode 100. We want to determine the equivalent circuit values at the fundamental frequency:

1.) Our quartz crystal has a nominal Series resonant frequency of 12MHz.
The first task is to measure the parallel Capacitance C_0. How can we do this ?

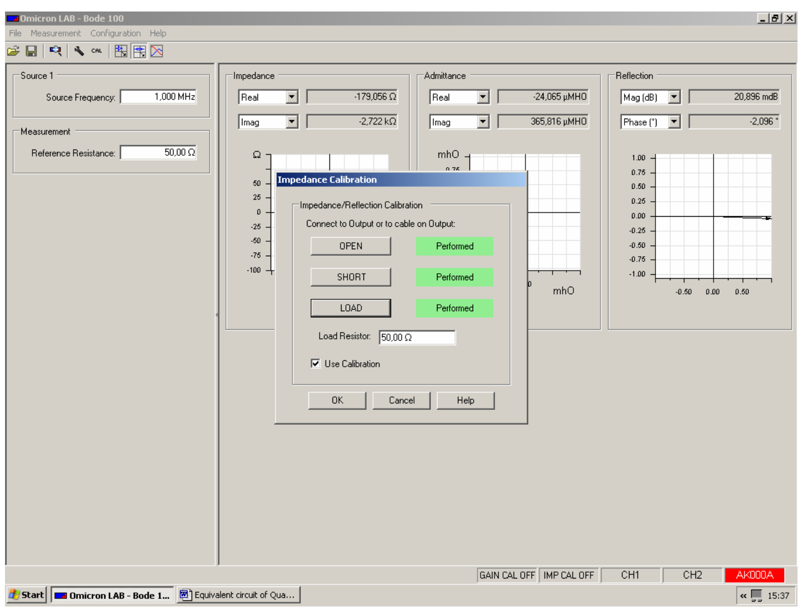
We use Bode 100 to measure the impedance of the crystal at a frequency which is well appart from the Series-Parallel resonant frequencies. For the 12MHz crystal 10MHz will be a good value. The result will be a nearly pure reactance of capacitive type. Before we start the measurement we have to set the measurement frequency, the measurement level and we have to perform an impedance calibration at the end of the connection cable used for the measurement.

2.) Start the Impedance/Reflection measurement, open the internal configuration window and make following settings:

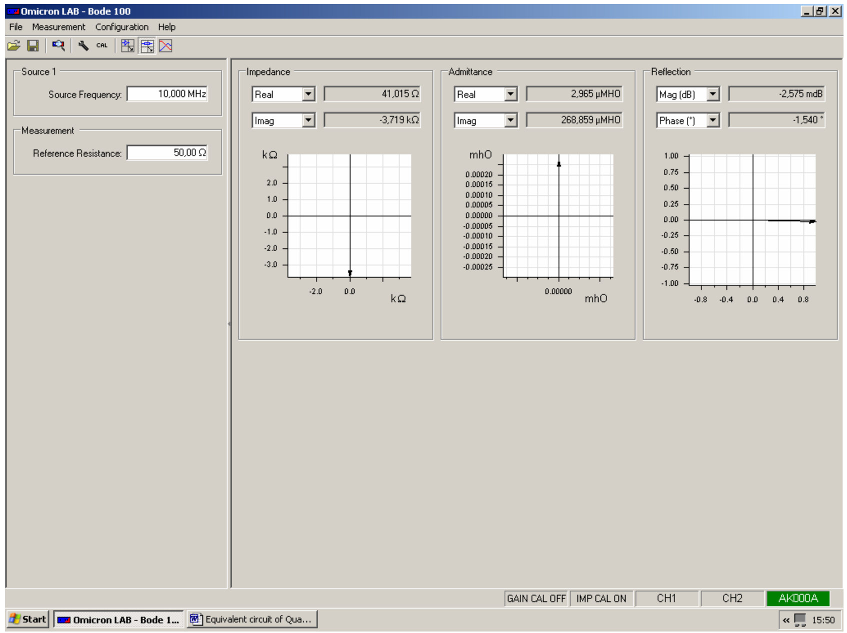
Source frequency:	10MHz
Receiver Bandwidth:	10Hz (minimum bandwidth to avoid measurement errors caused by noise)
Level:	0dBm (preset value)



3.) Open the Calibration window and perform the impedance calibration by measuring “Open”, “Short” and “Load” as described in the Bode 100 user manual.



4.) Now we are ready to measure the Impedance of the crystal. Connect the measurement cable to the “IN” BNC connector and the “Short” to the “OUT” BNC connector of the Quartz filter on the Test PCB. By using right click and pressing “Optimize” in each diagramm you will get the following display:



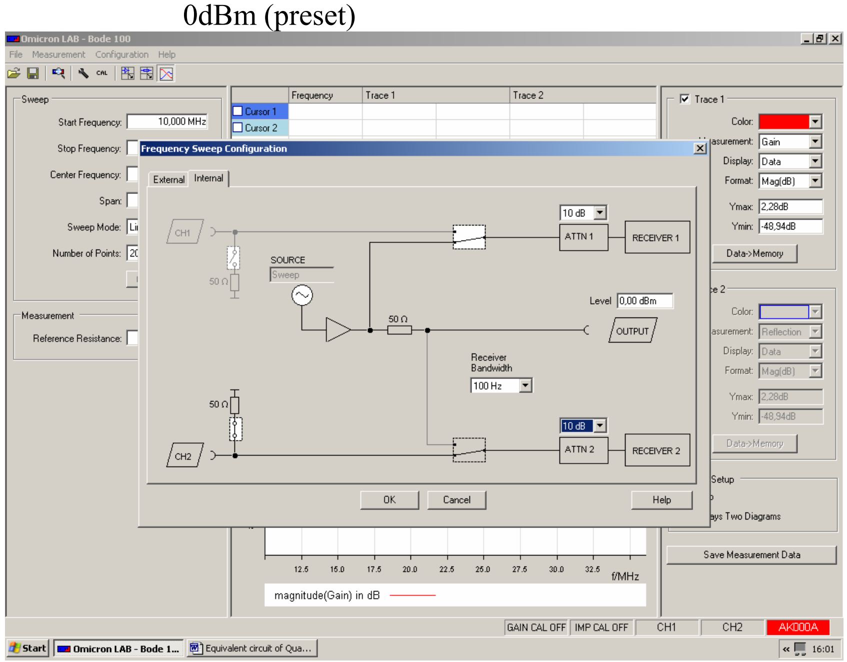
The readout for the Impedance is: $\underline{Z} = 41.015 - j\,3719$ Ohm at 10MHz
and with Xc = -3719 Ohm we get:

$$Co = \frac{1}{2\pi f\left|Xc\right|}$$

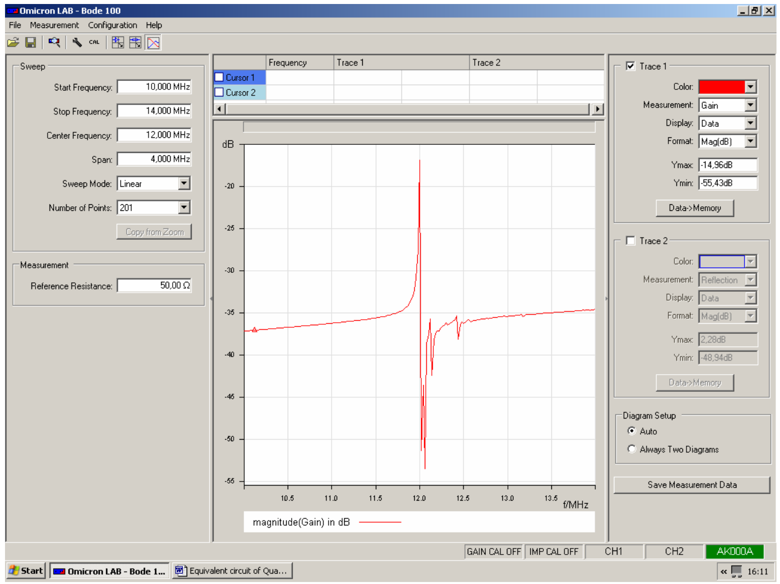
$$Co = 4.28\,pF$$

5.) Now we have to measure the fundamental Series and Parallel resonant frequencies of the crystal. For this purpose we start the frequency sweep Gain measurement, connect the crystal to OUTPUT and CH2 by means of 50 Ohm cables, open the internal configuration window and make following settings:

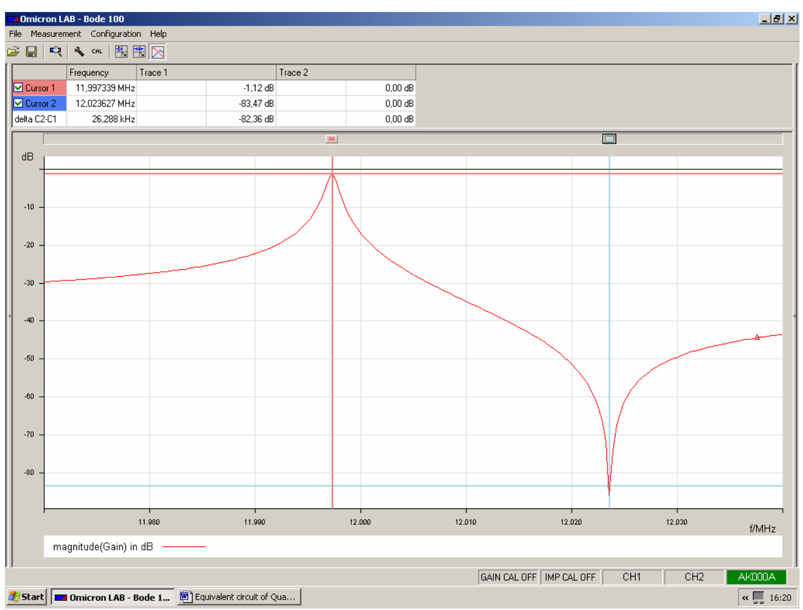
CH1 and CH2 attenuator:	10dB	Receiver Bandwidth:	100Hz
CH2 impedance:	50 Ohm	Internal reference (preset)	
Level:	0dBm (preset)		



6.) Leave the configuration window by pressing OK and make following settings in the Frequency sweep window:
Start 10MHz Stop 14MHz Switch off Trace 2
use preset settings for Trace 1 and Optimize to produce the following trace:



As we can see, the frequency span is too high, so we have to zoom in by right click and selecting Zoom mode. Zooming in and applying Copy from Zoom will produce a closer look to the interesting range. Finally you may adjust Start Frequency and Stop Frequency to convenient values – 11.97MHz and 12.04MHz for this measurement. To measure the resonant frequencies we move one cursor to the maximum and the other to the minimum of the displayed trace. The input fields are removed by double click an the separators.



Reading the cursor results we obtain the following frequencies:

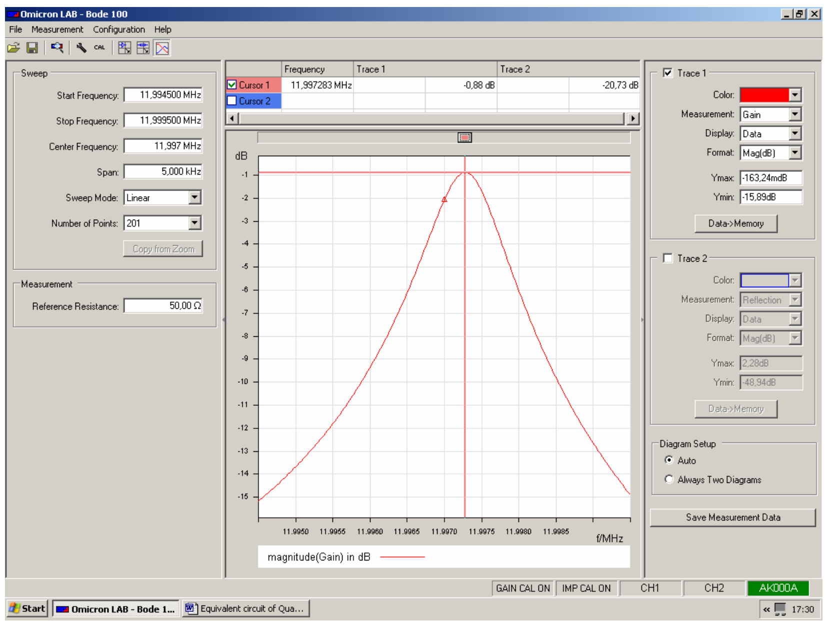
f_S = 11.997339MHz and f_P = 12.023627MHz

Now we can calculate the values for C_S and L_S of the crystal and we get:

$C_0 = 4.28pF$
$C_S = 9.37fF$
$L_S = 18.76mH$

7.) Finally we have to measure the Series Resistance R_S to calculate Q of the crystal. One possibilty is to use the measured attenuation at the Series resonant frequency, the other way is to measure the impedance of the crystal at the Series resonant frequency.

Restore the Input fields by double click on the separators and adjust the Center Frequency to 11.997339MHz and the Span to 5kHz, because now we only want to see the region around the Series resonant frequency. To increase the measurement accurracy we perform a gain – phase calibration and get the following result:



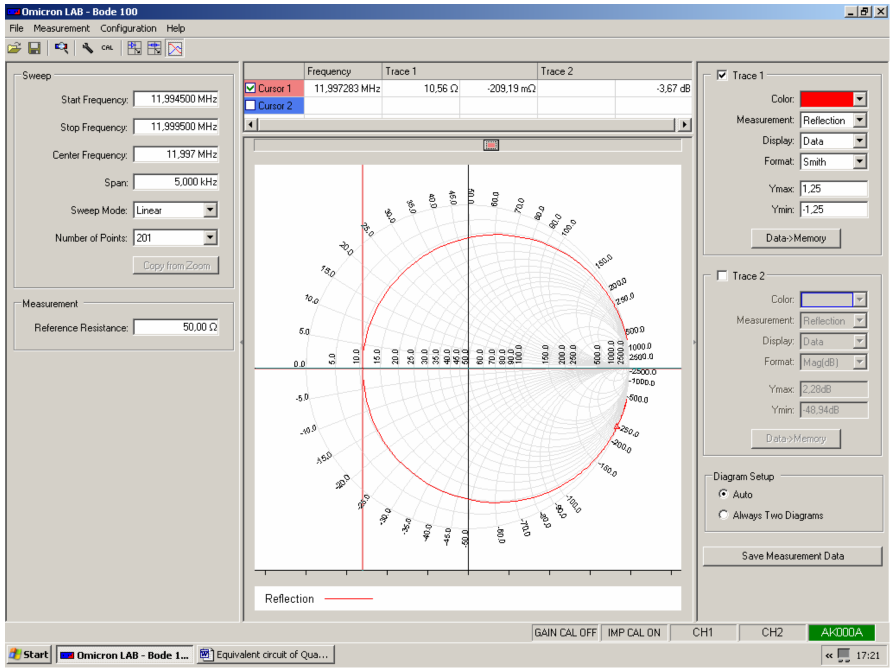
The readout of cursor 1 gives us an attenuation of 0.88dB. To calculate R_S we have to use the following equation:

$$Rs = 2 \cdot R \cdot \left(10^{\frac{a}{20}} - 1 \right)$$

$$Rs = 100 \cdot \left(10^{\frac{0.88}{20}} - 1 \right) = 10.66\ Ohm$$

a … attenuation in dB
R … Source resistance, Receiver resistance - 50 Ohm both for Bode 100

8.) To check our result obtained from the gain measurent, we perform a Reflection measurent. Select Reflection in the Measurement field and Smith chart in the Format field for Trace 1. Now we have to perform the impedance calibration in the sweep mode.
After calibration connect the measurement cable to the "IN" BNC connector and the "Short" to the "OUT" BNC connector of the Quartz filter on the Test PCB. The following trace should be displayed now:



Place cursor 1 to the left most point of the trace, which corresponds to the Series Resonant frequency. The value of R_S is 10.56 Ohm with this measurement.
The values of R_S are almost the same for both methods.
Now we have the values for all components of the equivalent circuit of the crystal and in addition we can calculate Q

$C_0 = 4.28pF$
$C_S = 9.37fF$
$L_S = 18.76mH$
$R_S = 10.56Ohm$
$Q = 133.916$