This is for
- Teachers in electronics
- Electronic Lab exercises

Lesson time: 3-4 Hours

Prerequisite: Understanding how crystals are described in an equivalent circuit.

High school				Technical School				University			
1st	2nd	3rd	4th	1st	2nd	3rd	4th	1st	2nd	3rd	4th

Lesson Plan: Measurement of the equivalent circuit of a quartz crystal

1 Introduction

1.1 Why should students listen to your quartz lesson?

Where is quartz used in modern electronics?
What do you think is the reason quartz is used so often?
Quartz is used in electronic circuits to get a high accuracy time standard. Accuracy you need in clocks, computers, controllers, game boys, toys and many more.

1.2 Target: After this lesson

You can answer following questions:
- What is the serial resonance?
- What is the parallel resonance?
- What is the anti-resonance?
- How does the Q range of quartz compare to a RLC resonance circuit?
- Why is the quartz frequency 32.768KHz so popular?

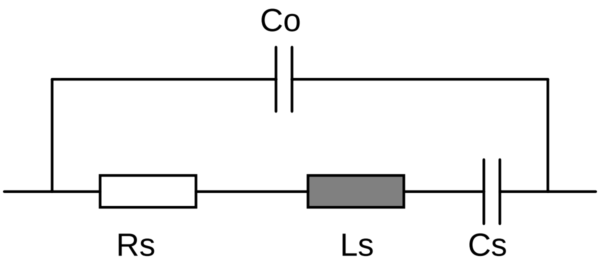
1.3 Your tasks after this lesson

After the lesson we want that you go home and check on the Internet what you can find about quartz. You will find some circuits & schemes so that you can see how they are implemented.

2 Main Part

2.1 Theory

An equivalent description of a quartz crystal is given by the following circuit. It is valid in the region of a single Series-Parallel resonance combination. As can be shown these combinations occur at odd multiples (odd overtones of the crystal) of the fundamental Series resonant frequency.



With known values of L_S, C_S, C_0 and R_S we obtain the following equations:

$$\textit{Series resonant frequency:} \quad fs = \frac{1}{2 \cdot \pi \cdot \sqrt{Ls \cdot Cs}}$$

$$\textit{Parallel resonant frequency:} \quad fp = fs \cdot \sqrt{1 + Cs / Co} \approx fs \cdot \left(1 + \frac{Cs}{2 \cdot Co}\right)$$

$$\textit{Quality factor at } fs: \quad Q = \frac{2 \cdot \pi \cdot fs \cdot Ls}{Rs} = \frac{1}{2 \cdot \pi \cdot fs \cdot Cs \cdot Rs}$$

As we don't know these values we have to solve the equations for L_S, C_S first, because we can measure C_0, f_S and f_P with the Bode 100.

$$Cs = \left(\frac{fp}{fs} - 1\right) \cdot 2 \cdot Co$$

$$Ls = \frac{1}{4\pi^2 fs^2 Cs}$$

Instead of using the above relationships it is possible to calculate the equivalent circuit out of a different set of measurements using the following equations :

$$\textit{Loaded Q at } fs: \quad Q_L = \frac{2 \cdot \pi \cdot fs \cdot Ls}{2 \cdot R + Rs} \quad \textit{with } R = 50\Omega \textit{ for a } 50\Omega \textit{ measurement setup}$$

$$\textit{Quality factor at } fs: \quad Q = Q_L \cdot \left(1 + \frac{2 \cdot R}{Rs}\right)$$

$$Ls = \frac{Q \cdot Rs}{2 \cdot \pi \cdot Ls} \qquad Cs = \frac{1}{Q \cdot 2 \cdot \pi \cdot fs \cdot Rs}$$

2.2 Measurement of C_O of the crystal:

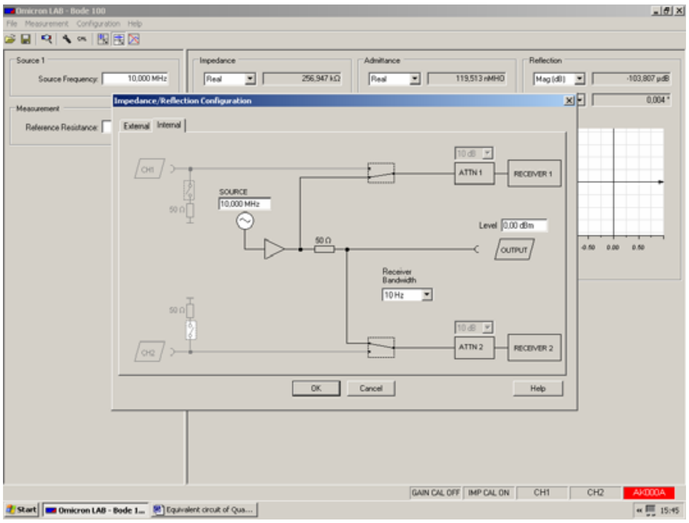
The Device Under Test is the crystal mounted at the Test PCB delivered with the Bode 100. We want to determine the equivalent circuit values at the fundamental frequency:

Our quartz crystal has a nominal Series resonant frequency of 12MHz.
The first task is to measure the parallel Capacitance C_0. How can we do this?

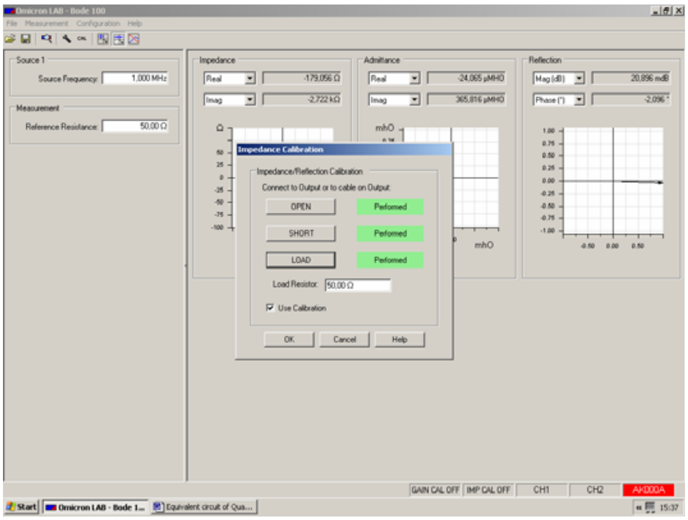
We use Bode 100 to measure the impedance of the crystal at a frequency that is well appart from the Series-Parallel resonant frequencies. For the 12MHz crystal 10MHz will be a good value. The result will be a nearly pure reactance of capacitive type. Before we start the measurement we have to set the measurement frequency, the measurement level and we have to perform an impedance calibration at the end of the connection cable used for the measurement.

Start the Impedance/Reflection measurement, open the internal configuration window and make following settings:

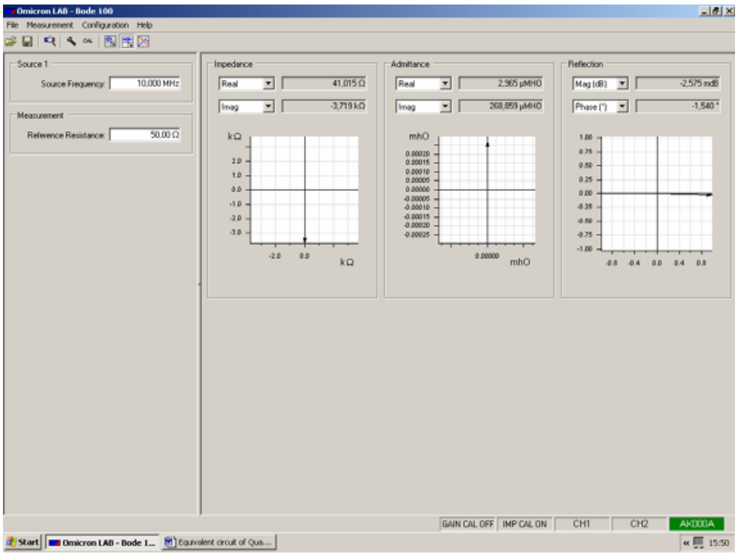
Source frequency:	10MHz	
Receiver Bandwidth:	10Hz	(minimum bandwidth to avoid measurement errors caused by noise)
Level:	0dBm	(preset value)


Source Frequency: 10,000 MHz
Impedance: Real 256,947 kΩ
Admittance: Real 119,513 nMHO
Reflection: Mag (dB) -103,807 µdB
0,004 °
SOURCE 10,000 MHz
Level 0,00 dBm
Receiver Bandwidth 10 Hz

Open the Calibration window and perform the impedance calibration by measuring “Open”, “Short” and “Load” as described in the Bode 100 user manual.


Source Frequency: 1,000 MHz
Reference Resistance: 50,00 Ω
Impedance: Real -179,056 Ω; Imag -2,722 kΩ
Admittance: Real -24,065 µMHO; Imag 365,816 µMHO
Reflection: Mag (dB) 20,896 mdB; Phase (°) -2,096 °
OPEN Performed
SHORT Performed
LOAD Performed
Load Resistor: 50,00 Ω
☑ Use Calibration

Now we are ready to measure the Impedance of the crystal. Connect the measurement cable to the “IN” BNC connector and the “Short” to the “OUT” BNC connector of the Quartz filter on the Test PCB. By using right click and pressing “Optimize” in each diagram you will get the following display:


Omicron LAB - Bode 100

Source 1
Source Frequency: 10.000 MHz

Measurement
Reference Resistance: 50.00 Ω

Impedance
Real 41,015 Ω
Imag -3,719 kΩ

Admittance
Real 2,965 µMHO
Imag 268,859 µMHO

Reflection
Mag (dB) -2,575 mdB
Phase (°) -1,540 °

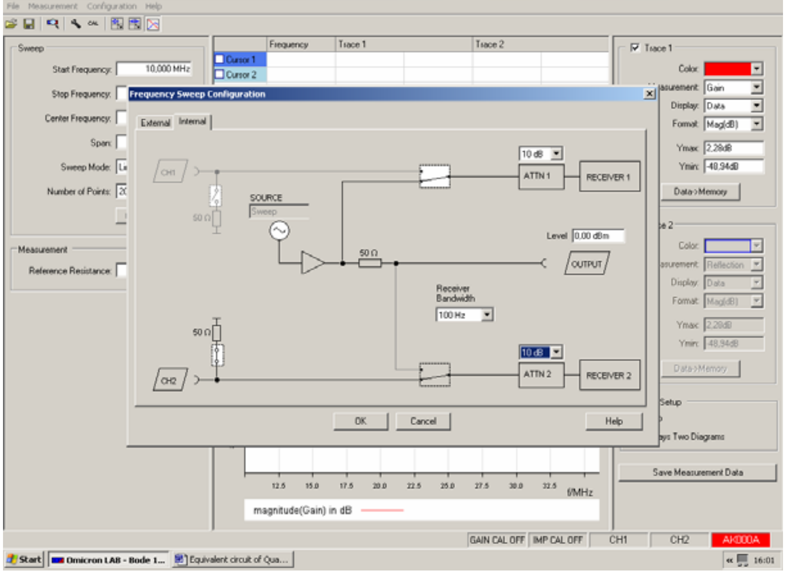
The readout for the Impedance is: $\underline{Z} = 41.015 - j\,3719$ Ohm at 10MHz
and with Xc = -3719 Ohm we get:

$$Co = \frac{1}{2\pi f\left|Xc\right|}$$

$$Co = 4.28\,pF$$

2.3 Measurement of the loaded Q and Rs of the crystal:

Now we have to measure the loaded Q at fs and the Series resistance of the crystal.
For this purpose we start the frequency sweep Gain measurement, connect the crystal to OUTPUT and CH2 by means of 50 Ohm cables, open the internal configuration window and make following settings:
CH1 and CH2 attenuator: 10dB
Receiver Bandwidth: 100Hz
CH2 impedance: 50 OhmInternal reference (preset)
Level: 0dBm (preset)


Omicron LAB - Bode 100

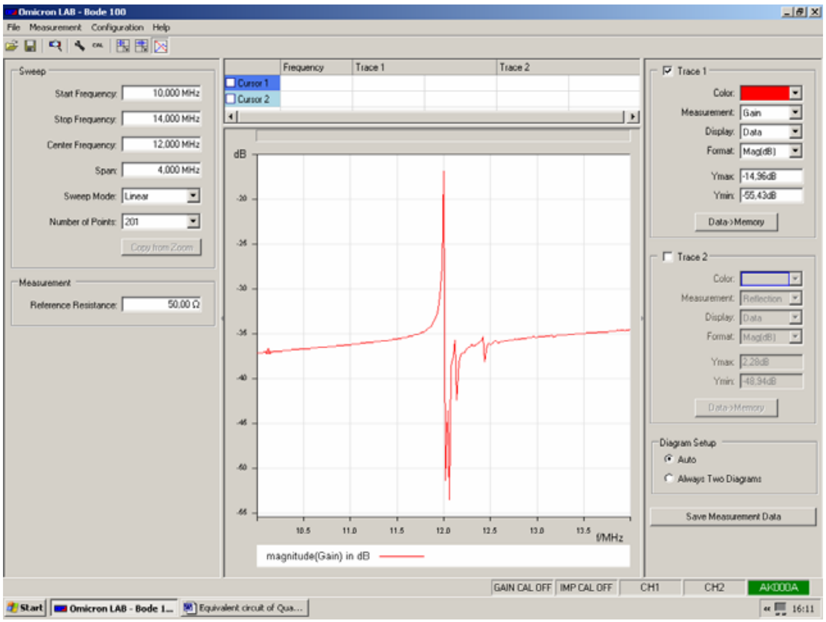
Sweep
Start Frequency: 10.000 MHz

Frequency Sweep Configuration
External | Internal
ATTN 1: 10 dB
ATTN 2: 10 dB
Level: 0.00 dBm
Receiver Bandwidth: 100 Hz
SOURCE: Sweep

☑ Trace 1
Measurement: Gain
Display: Data
Format: Mag(dB)
Ymax: 2,28dB
Ymin: -48,94dB

Trace 2
Measurement: Reflection
Display: Data
Format: Mag(dB)
Ymax: 2,28dB
Ymin: -48,94dB

Leave the configuration window by pressing OK and make following settings in the Frequency sweep window:
Start 10MHz
Stop 14MHz
Switch off Trace 2
use preset settings for Trace 1 and Optimize to produce the following trace:



Sweep
Start Frequency: 10,000 MHz
Stop Frequency: 14,000 MHz
Center Frequency: 12,000 MHz
Span: 4,000 MHz
Sweep Mode: Linear
Number of Points: 201

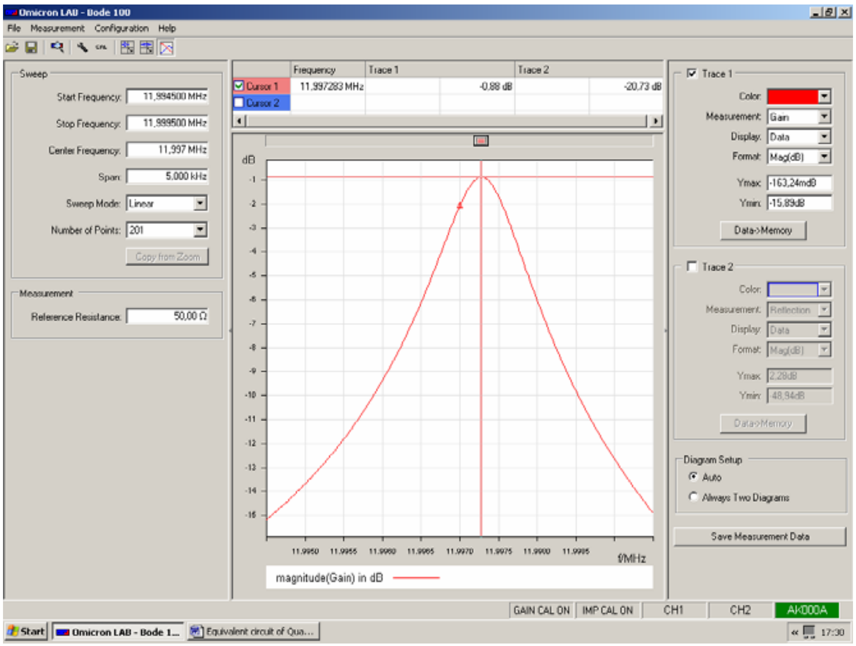
Measurement
Reference Resistance: 50,00 Ω

☑ Trace 1
Measurement: Gain
Display: Data
Format: Mag(dB)
Ymax: -14,96dB
Ymin: -55,43dB

☐ Trace 2
Measurement: Reflection
Display: Data
Format: Mag(dB)
Ymax: 2,28dB
Ymin: -48,94dB

Diagram Setup
● Auto
○ Always Two Diagrams

As we can see, the frequency span is too high, so we have to zoom in by right click and selecting Zoom mode. Zooming in and applying Copy from Zoom will produce a closer look to the interesting range around fs.



Sweep
Start Frequency: 11,994500 MHz
Stop Frequency: 11,999500 MHz
Center Frequency: 11,997 MHz
Span: 5,000 kHz
Sweep Mode: Linear
Number of Points: 201

Measurement
Reference Resistance: 50,00 Ω

	Frequency	Trace 1	Trace 2
☑ Cursor 1	11,997283 MHz	-0,88 dB	-20,73 dB
☐ Cursor 2			

☑ Trace 1
Measurement: Gain
Display: Data
Format: Mag(dB)
Ymax: -163,24mdB
Ymin: -15,89dB

☐ Trace 2
Measurement: Reflection
Display: Data
Format: Mag(dB)
Ymax: 2,28dB
Ymin: -48,94dB

Diagram Setup
● Auto
○ Always Two Diagrams

Using the cursor 1 and cursor 2 we get fs = 11.997339MHz and B_{3dB} = 978Hz and Q_L = 12267
Finally we have to measure the Series Resistance R_S to calculate Q of the crystal. One possibilty is to use the measured attenuation at the Series resonant frequency, the other way is to measure the impedance of the crystal at the Series resonant frequency.

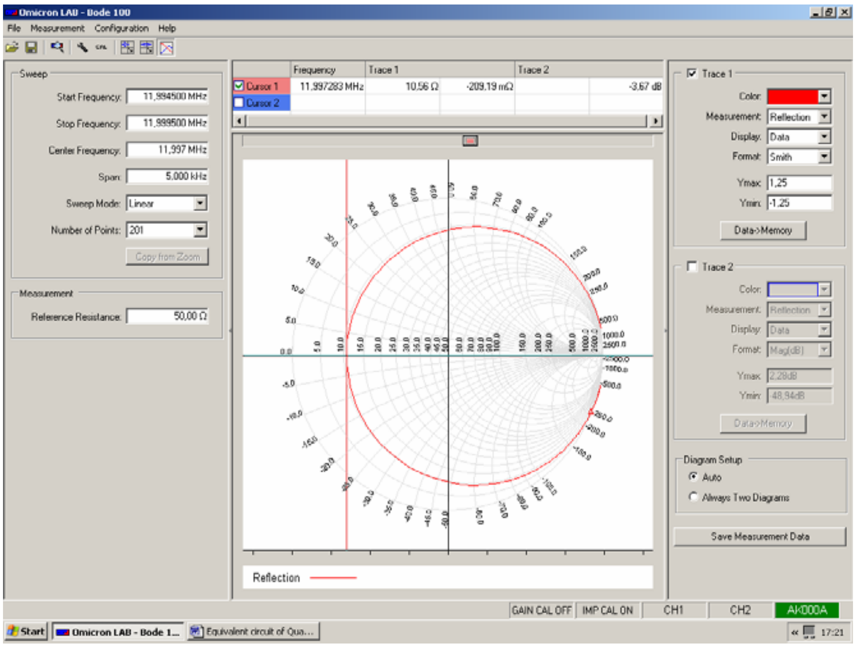
The readout of cursor 1 gives us an attenuation of 0.88dB. To calculate R_S we have to use the following equation:

$$Rs = 2 \cdot R \cdot \left(10^{\frac{a}{20}} - 1 \right)$$

$$Rs = 100 \cdot \left(10^{\frac{0.88}{20}} - 1 \right) = 10.66\ Ohm$$

a … attenuation in dB
R … Source resistance, Receiver resistance : 50 Ohm both for Bode 100

To check our result obtained from the gain measurent, we perform a Reflection measurent. Select Reflection in the Measurement field and Smith chart in the Format field for Trace 1.
Now we have to perform the impedance calibration in the sweep mode.
After calibration connect the measurement cable to the “IN” BNC connector and the “Short” to the “OUT” BNC connector of the Quartz filter on the Test PCB. The following trace should be displayed now:



Sweep
Start Frequency: 11,994500 MHz
Stop Frequency: 11,999500 MHz
Center Frequency: 11,997 MHz
Span: 5,000 kHz
Sweep Mode: Linear
Number of Points: 201

Measurement
Reference Resistance: 50,00 Ω

	Frequency	Trace 1		Trace 2
☑ Cursor 1	11,997283 MHz	10,56 Ω	-209,19 mΩ	-3,67 dB
☐ Cursor 2				

☑ Trace 1
Color: 🟥
Measurement: Reflection
Display: Data
Format: Smith
Ymax: 1,25
Ymin: -1,25

☐ Trace 2
Measurement: Reflection
Display: Data
Format: Mag(dB)
Ymax: 2,28dB
Ymin: -48,94dB

Diagram Setup
● Auto
○ Always Two Diagrams

Place cursor 1 to the left most point of the trace, which corresponds to the Series Resonant frequency.
The value of R_S is 10.56 Ohm with this measurement.
The values of R_S are almost the same for both methods.

2.4 Calculation of the complete equivalent circuit of the crystal

First we have to calculate the quality factor of the crystal itself (unloaded Q).
With Q_L = 12267, Rs = 10.6 Ohm and R = 50 Ohm we get Q = 127993

C_0 = 4.28pF
Rs = 10.6Ohm
Ls ≈ 18mH
Cs ≈ 9.8fF
Q ≈ 128000

2.5 Some numbers and facts of crystal oscillators in different applications

- Microcontroller applications, frequency range from 1MHz up to 50MHz, accuracy uncritical about ± 200ppm, frequency deviation at 10MHz less than 2kHz
- Transmitter applications in cable head ends, often 10MHz oscillators as reference frequency for phase locked loops, these oscillators are temperature compensated to obtain better frequency stability, accuracy better than ±10ppm, frequency deviation at 10MHz less than 100Hz
  this means that the frequency deviation of a 10MHz oscillator is lower than 100Hz !
- Standard transmitter applications, often 10MHz oscillators as reference frequency for phase locked loops, these oscillators heated to temperatures in the range between 60°C and 90°C to obtain better frequency stability, accuracy better than ±1ppm
  this means that the frequency deviation of a 10MHz oscillator is lower than 10Hz !
- Oscillators used as frequency standard, often combined with references as GPS or other frequency standards, these crystal oscillators are heated in double control loops and reach frequency deviations of less than 10mHz/day
- Oscillators in clocks – the frequency is 32.768Hz. This frequency is divided by 2^{15} (binary divider) and we get one pulse per second. If we suppose that the oscillator has an accuracy of 10ppm our clock will deviate 5 minutes in one year.
- Some pictures of crystals